Math 315-003 Final Exam Name____________________
April 20, 2004 Show relevant work!
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1. Explain what if means for a function $f : D \to \mathbf{R}$ to be continuous.
   a. Give the sequence definition.

   b. Give the epsilon-delta definition.

2. Show a bounded increasing sequence converges to its supremum.

3. State and prove the (Lagrange) Mean Value Theorem. State your assumptions.

4. Define uniform continuity and prove that a continuous function on a closed interval is uniformly continuous.

5. State and prove the First Fundamental Theorem of Calculus.

6. Suppose $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are series of positive numbers such that $\lim_{k\to\infty}\left(\frac{a_k}{b_k}\right) = \lambda$ and $\lambda > 0$. Show that $\sum_{k=1}^{\infty} a_k$ converges if and only if the series $\sum_{k=1}^{\infty} b_k$ converges.

7. For a number r such that $|r| < 1$, show $\sum_{k=0}^{\infty} r^k$ converges.

8. Give an example of a sequence of functions $f_n : [0,1] \to \mathbf{R}$ the converges pointwise but not uniformly. Prove your assertions.

9. Suppose that $f_n : D \to \mathbf{R}$ is a sequence of continuous functions that converges uniformly to the function $f : D \to \mathbf{R}$. Show the limit function $f : D \to \mathbf{R}$ is also continuous.

10. Find the Taylor series for $\tan^{-1} x$ and give its interval of convergence where it converges to $\tan^{-1} x$. Hint: $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$.